STUDYING MATHEMATICS AT THE UNIVERSITY: THE INFLUENCE OF LEARNING STRATEGIES

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Many students studying mathematics at university experience great difficulties in their first year. In Germany, universities have to deal with drop-out rates of 40-50% of first-year students with a major in mathematics. Possible reasons are differences between school and university concerning the character of mathematics taught and the learning culture. In a study with N = 118 students with a major in mathematics, we investigate individual determinants of students' competency development in their first semester. First results indicate that the type of learning strategies mainly used by students has an impact on mathematics achievement, interest and self-concept. Except for the case of high achieving students, these types of learning strategies are not related to individual variables at the beginning of the study.

INTRODUCTION

The transition from school to university is experienced as an interesting and exciting period for many first-year students. This transition is challenging for students studying mathematics due to requirements including a significant time investment, self-organized learning, and academic mathematics itself. Many students fail to cope with these unexpected challenges when starting at the university. Hence, during the last decades these specific challenges were investigated from different perspectives focusing on the differences between mathematics at school and academic mathematics and the corresponding learning processes at the university (e.g., de Guzmán, Hodgson, Robert, & Villani, 1998; Hoyles, Newman, & Noss, 2001).

In Germany, universities and mathematics departments are faced with comparatively high drop-out rates (up to 50%) of first-year students in mathematics. According to surveys, students report that this is mainly caused by the enormous pressure to perform and the lack of motivation (Heublein et al., 2009). For a deeper insight into these reasons, we started a research project on the development of students' competencies and the teaching and learning processes during the first semester. In this contribution, we focus on first results concerning individual cognitive and non cognitive variables and their impact on the learning of mathematics in the first six weeks of an Analysis I course (Analysis I encompass real analysis and has a theory-based level emphasizing definitions, theorems and proofs. Calculations like in calculus courses play only a minor role).

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THEORETICAL BACKGROUND

Based on a review of research literature, we postulate that the difficulties of first-year students in mathematics are mainly rooted in two fundamental differences between mathematics learning at school and mathematics learning at university: (1) the character of mathematics that is taught and (2) the individual learning strategies necessary to use the learning opportunities effectively.

Character of Mathematics at School and at University

Many mathematics educators agree that mathematics taught in high school is not just academic mathematics in a simplified form but has its own character (e.g., Hoyles, et al., 2001). The main reason is that learning mathematics in high school must contribute to the aim of general education. This means, in particular, that students should learn how to use mathematics for solving everyday problems. Accordingly, in school curriculum there is a specific emphasis on mathematical content which is relevant for the application of mathematics as tool (e.g., percentages, algebraic manipulations) but which is hardly interesting from a scientific mathematical perspective (e.g., Dörfler, & McLone, 1986). At university, students with a major in mathematics learn mathematics as a scientific discipline. This means that the mathematical content is organized and presented in a specific axiomatic and rigorous manner. In the first semesters, applications of mathematics for solving real-world problems hardly play any role.

Hoyles et al. (2001, p. 841) characterizes these two sides of mathematics: "It is a tool in the study of the sciences, and it is an object of study in its own right." These two sides of mathematics are reflected in high school and at university in quite a different way which has serious consequences for the role of important characteristics of mathematics like proving, rigor or formalism. For example, most of the mathematical concepts in school are encountered in an informal way (Engelbrecht, 2010) so that students develop and work with a *concept image* (Tall & Vinner, 1981). A formal *concept definition* frequently does not play a prominent role. At university, it is just the opposite: concepts are introduced by a formal definition which is necessary to meet the standards of rigor. In the case of mathematical proof, a similar situation can be observed. Proofs are essential when dealing with mathematics as a scientific theory because they give evidence for statements and explain internal relations. Considering mathematics from an instrumental perspective (as a tool), proofs play a minor role and they are often omitted (it is enough to know that there exists a proof).

Teaching and Learning Mathematics at School and at University

The teaching and learning of mathematics at school and at university differs in two important aspects. On the one hand, the formal organization of learning opportunities and, on the other hand, the individual learning strategies necessary for an effective use of the learning opportunities. At German universities, the Analysis courses for first-year students consist of three different learning opportunities a week: two 90 minute lectures given by a mathematics professor, a set of 3-5 challenging exercises as obligatory homework (self-study phase in small study groups) and a 90 minutes tutorial per week where a senior mathematics student discusses the solutions of the homework with a group of 20-30 students. In particular, the self-study phase during which the students work in small groups on their homework is considered an important learning activity since students are individually involved in mathematical problem solving processes.

Regarding effective learning strategies at school and at university we can observe a necessity of additional learning strategies at the university essential for successful competence acquisition. These learning strategies correspond with the fact that mathematics at university is taught as scientific discipline. First, students need to apply elaboration strategies to understand the formally presented mathematical content in the lectures. New mathematical concepts cannot be grasped through formal concept definitions, so it is necessary that students connect the concept definition to an already existing concept image from an intuitive use of this concept in school (e.g., in the case of limits) or that they individually develop a new concept image (Engelbrecht, 2010). Second, students need to elaborate and reflect on problem-solving strategies. As Dreyfus (1991) criticizes, mathematics is taught as a completed theory and students are not involved in the trial and error process of creating new knowledge. In particular, problem-solving strategies are kept implicitly in the lectures. Accordingly, students do not get a model how to approach proof problems. Based on research on example-based learning, self-explanation has to be proven as an effective learning strategy (e.g., Chi et al., 1989). Therefore, one can assume that students performing self-explanations in the self-study phase when working through problem solutions or proofs show a better achievement than students who only comprehend solutions or proofs without self-explanations.

RESEARCH QUESTIONS AND DESIGN OF THE STUDY

Based on the theoretical background, our subsequently presented study is guided by the following research questions:

- What are the individual learning prerequisites of students starting to study mathematics? Here we address the final school grade (overall and in mathematics), the prior knowledge in analysis, interest in mathematics and mathematics self-concept.
- How do the individual interest in mathematics and the mathematics self-concept develop in the first six weeks?
- What type of learning strategies do students mainly apply in self-study phase (homework, self-organized study groups) after six weeks?
- What impact do the learning strategies have on the achievement, interest in mathematics and the mathematics self-concept?

- Is the use of the learning strategies influenced by the individual learning prerequisites students bring from school?

Sample and Methodology

Our on-going study is conducted at the Department of Mathematics of the University of Kiel (Germany) from October 2010 to February 2011 (winter term). The sample consists of 118 students majoring in mathematics which started in October 2010.
On the first day of the winter term in October 2010 we collected data for learning prerequisites consisting of the final school grade (overall and in mathematics), the prior knowledge in the field of analysis, interest in mathematics and mathematics self-concept. In December 2010, after six weeks, we collected data for achievement in analysis, interest in mathematics, mathematics self-concept and learning strategies applied during self-study phases.
We used approved questionnaires adapted from Schiefele, Moschner, and Husstegge (2002) for measuring interest in mathematics (6 items, Cronbach's α = .83) and mathematics self-concept (4 items, α = .80). On both questionnaires students had to rate statements on a four-point Likert scale (0 = strongly disagree, 1 = disagree, 2 = agree, 3 = strongly agree). The tests for prior knowledge in the field of analysis and the analysis achievement test consist of 10 and 12 items (α = .63 and α = .73) respectively (open items as well as multiple choice items, see appendix for two sample items). Information about the learning strategies the students mainly applied was collected by a questionnaire. Students should report which of the following three learning types fits best to their own behaviour in self-study phases:

- "I study the exercises intensively and try to solve them. I try to comprehend the solutions of other students. I give rarely self-explanations." (reproduction type)
- "I study the exercises intensively and try to solve them. I try to comprehend the solutions of other students. I explain the solution to myself and/or to other students even if I rarely find solutions by myself." (self-explanation type)
- "Often, I can solve the exercises or I find ideas for solutions. Then I explain the solution to myself and/or other students." (self-solver type)

RESULTS

The data for the learning prerequisites indicate that the mathematics students start university with quite good prerequisites. The mean value for their final school grade in mathematics is *M* = 12.0 (*SD* = 2.13) of maximal 15 points and the means for interest in mathematics *M* = 2.17 (*SD* = 0.41) and mathematics self-concept *M* = 1.91 (*SD* = 0.45) are comparatively high. The mean value of the overall final school grade reflects only a moderate level: *M* = 2.22 (*SD* = 0.55) where 1.0 is the best and 4.0 is the worst possible grade. It seems that they have a specific strength in mathematics.

Interestingly, the results for the test on prior knowledge in analysis are comparatively weak with *M* = 4.64 (*SD* = 2.16) of maximal 10 points. Since this test asks for knowledge which is part of the school curriculum but is not really emphasized in school, this weak results supports the assumption that mathematics in school and academic mathematics have a different character. Consistent with this finding is the fact that the average value for students' mathematics self-concept decrease significantly in the first six weeks to $M = 1.59$ ($SD = 0.54$, $t(117) = 7.94$, $p < .001$, $d = 0.66$). It seems that in this period the students become aware that learning mathematics at university is much more challenging than they were used to from their school experience. Regarding the interest in mathematics, the mean values remain on a comparatively high level ($M = 1.98$, $SD = 0.51$, $t(117) = 4.64$, $p < .001$, $d = 0.41$). Although there is also a decrease, the mean value is still on the "agree"-level (= 2.00).

As described previously, data for the applied learning strategies in self-study phases were collected by a self-report. About 35% of the students think that their learning strategies fits best to the reproducing type, whereas 52% choose the self-explanation type and 13% the self-solver type. Based on the empirical evidence from research on example-based learning, we expect that students of the self-explanation type show a better achievement after six weeks of the winter term than students of the reproducing type (e.g. Chi et al., 1989). Obviously, students of the self-solver type are expected to be stronger than both other learning strategies types.

Results of an ANCOVA with prior knowledge in analysis as covariate indicate that the type of learning strategies have an significant impact on the achievement in analysis after six weeks (see Table 1, $F(2,114) = 38.03$, $p < .001$, $R^2 = .49$). However, post-hoc tests show that indeed students of the self-solver type are significantly better than the other two types ($p < .001$) but there are no significant differences between students of the reproducing type and of the self-explanation type ($p = .743$).

Mean (max. 12 pts.)	Reproducing Type	Self-Explanation Type	Self-Solver Type
Achievement in Analysis	4.93	5.41	8.31

Table 1. Achievement in analysis after six weeks, estimated marginal means with prior knowledge in analysis as covariate.

Regarding the development of interest in mathematics and mathematics self-concept, students of the self-solver type show increasing (interest) or stable (self-concept) values (see Table 2). For students using learning strategies of the other two types, interest and self-concept decreases within the first six weeks. Here it is remarkable that students of the reproducing type show a significant lower self-concept after six weeks than students of the self-explanation type ($M = 1.31$ ($SD = 0.46$) and $M = 1.61$ ($SD = 0.48$), $t(100) = -3.12$, $p = .002$, $d = -0.63$ for self-concept).

M (SD)	Reproducing Type		Self-Explanation Type		Self-Solver Type	
	First day	after 6 weeks	First day	after 6 weeks	First day	after 6 weeks
Interest in Mathematics	2.05 (0.43)	1.75 (0.48)	2.23 (0.38)	2.01 (0.46)	2.23 (0.41)	2.44 (0.39)
	$t(40) = 4.65$, $p < .001$, $d = 0.65$		$t(60) = 3.88$, $p < .001$, $d = 0.52$		$t(15) = -2.83$, $p = .013$, $d = -0.52$	
Mathematics Self-concept	1.77 (0.46)	1.31 (0.46)	1.93 (0.37)	1.61 (0.48)	2.23 (0.54)	2.23 (0.41)
	$t(40) = 6.71$, $p < .001$, $d = 1.00$		$t(60) = 6.04$, $p < .001$, $d = 0.76$		n.s.	

Table 2. Development of interest in mathematics and mathematics self-concept in the first six weeks of the first semester divided by learning strategy types.
Likert-Scale: 0 = strongly disagree, 1 = disagree, 2 = agree, 3 = strongly agree

Since the type of learning strategy has an impact on achievement in analysis, on mathematics self-concept and on interest in mathematics after six weeks, it is interesting if the learning strategies are influenced by learning prerequisites students bring from school or if they were developed independently from these during the first six weeks of the winter term. It turned out that there are no significant differences between students of the reproducing type and students of the self-explanation type regarding the final school grade (overall and mathematics), prior knowledge in analysis, mathematics self-concept and interest in mathematics. However, students of the self-solver type differ significantly from the other types in their prior knowledge in analysis and in the mathematics self-concept already at the beginning of their study. It seems that this small group ($N = 16$) starts with better prerequisites.

DISCUSSION

Our findings indicate that first-year students with a major in mathematics at German universities can be characterized as students with a good school achievement in mathematics, high interest in mathematics and quite a good self-concept in mathematics. However, already after six weeks experience in studying mathematics, many students show a decreasing self-concept which is probably caused by the big challenges that go along with the transition phase from school to university. As we elaborated in the theoretical background, we assume that coping with these challenges requires specific learning strategies which differ from that in school. In particular, the ability to apply elaboration strategies like self-explanation (or explanation to others) is necessary to understand the presented scientific mathematics and to learn from sample solutions or other sources. As presented in the results section, the self-reported learning strategy type of the students has indeed a significant influence on the achievement in analysis after six weeks. Interestingly, we do not find the expected achievement differences between students of a reproducing learning type and students of a self-explanation type (but they differ significantly in their mathematics self-concept). Here, we assume that the self-reported learning strategy type is open for social desirability effects so that students may report that their learning strategies are on a higher level than it is really the case.

A noticeable result is that those students mainly using reproducing or self-explanation learning strategies after six weeks do not differ in their learning prerequisites at the beginning of their study. It seems that the type of learning strategies mainly used in self-study phases are not determined by the individual learning prerequisites. So, there should be good chances for an intervention teaching specific learning strategies in the first weeks at the university which support students to cope with the challenges of academic learning in mathematics.

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APPENDIX

Translated sample item of the test of prior knowledge in the field of analysis:

Does a smallest positive number exist? Which of the following statements is true?

1. *Yes, because you can find a number in IR, arbitrarily close to 0.*
2. *No, because for every positive number there exists another number between 0 and this number.*
3. *Yes, because the positive real numbers are bounded below.*
4. *No, because the smallest positive number is not real, but rational.*

Translated sample item of the analysis achievement test after six weeks:

Prove that for all $s \in IR$ *with* $s > 2$ *the series* $\sum_{k=1}^{\infty} \frac{1}{k^s}$ *is convergent.*